Models for Integrating Interest Rate, Sovereign Exchange Rate, and Credit Risk

ISI Technical Note

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A. Exchange Rate Risk

Sovereign exchange rate between two currencies depends on the evolution of the numeraire interest rate r_n and the sovereign interest rate r_g. The exchange rate X is a scaling factor that changes one unit of numeraire currency to one unit of sovereign currency. Its evolution usually modeled as a single factor stochastic differential equation:

$$dX/X = [r_n(t) - r_g(t)]\, dt + s\, dZ$$

Here s is a constant that corresponds to the volatility of the exchange rate.

Let $B(t,T)$ denote the price of a riskless zero-coupon bond with maturity T. The price of a sovereign zero-coupon bond with maturity T, priced in numeraire units is the risk neutral expectation:

$$V(t, T) = \mathbf{E[}X(t) * B(t, T) \mid r_n(t) \text{ and } r_g(t)\ \mathbf{]}$$

Note that the interest rate paths for $X(t)$, $r_n(t)$ and $r_g(t)$ are all required to be known.

B. Credit Risk

The price of a credit risky bond in a numeraire currency is correlated to the price of a similar riskless bond, but the price is usually lower due to credit ratings, default probabilities, and recovery rates..

One model for credit risky bonds was proposed by Jarrow, Landow, and Turnbull. The probability distribution for the time of default (given a particular credit rating) is modeled by a discrete time, time-homogeneous finite state space Markov chain, with transition probability matrix P. The states correspond to credit ratings (such as Aaa, B, C, D) that are published by credit rating agencies such as Moody's or S&P. The transition probability matrix is a rule that shows how ratings migrate over unit time intervals. The n-step transition probabilities, computed by P^n, are used to compute $Q(t,T)$ — the default probability after time T at time t and $d(t)$ — the recovery rate at time t.

When credit risky bonds default, they may receive a certain percentage of the bond value at the maturity of the bond (instead of zero). This percentage is the recovery rate. Recovery rates are also published by credit agencies.

Both default probabilities (see Kealhofer et al p. 45) and recovery rates (see *CreditMetrics Technical Document* p. 78-79) are random variables. Consequently, they both have means and standard deviations and may or may not be correlated with the probability distribution for the interest rates.

Suppose $B(t,T)$ is the price of a riskless zero-coupon bond with maturity T in a numeraire currency. Let $W(t, T)$ denote the price of a credit risky bond with maturity T. If $d(t)$ is the recovery rate at time t, and $Q(t,T)$ is the default probability after time T at time t, then its value is the risk neutral expectation:

$$W(t, T) = \mathbf{E[}\ B(t,T)\ [d(t) + (1 - d(t))\ Q(t,T)] \mid r_n(t), d(t), Q(t,T)\ \mathbf{]}$$

If $X(t)$ is an exchange rate at time t (subject to its specified stochastic differential equation), then the price of a credit risky sovereign bond is the risk neutral expectation:

$$Y(t, T) = \mathbf{E[}\ X(t)\ B(t,T)\ [d(t) + (1 - d(t))\ Q(t,T)]\ r_n(t), r_g(t), d(t), Q(t,T)\ \mathbf{]}$$